

Gauge fields at the surface of topological insulators

M. I. Katsnelson

*Radboud University Nijmegen, Institute for Molecules and Materials,
Heyendaalseweg 135, NL-6525 AJ Nijmegen, The Netherlands*

F. Guinea and M. A. H. Vozmediano

Instituto de Ciencia de Materiales de Madrid (CSIC), Cantoblanco, Madrid 28049, Spain

(Dated: January 26, 2013)

We study the emergence of gauge couplings in the surface states of topological insulators. We show that gauge fields arise when a three dimensional strong topological insulator is coated with an easy-plane ferromagnet with magnetization parallel to the surface. We analyze the modification induced by the gauge fields on the surface spectrum for some specific magnetic configurations.

PACS numbers:

INTRODUCTION

The Dirac fermion description of novel condensed matter systems have given rise to a great deal of new phenomena and ideas in the field that is acting as a bridge between different areas of physics including quantum field theory, cosmology, elasticity, and statistical mechanics.

One of the most successful examples can be found in the physics of graphene [1] and, in particular, in the attractive relation between the structural - lattice - properties and the electronics of this material. It is by now well known that elastic deformations and curvature of the graphene lattice give rise to effective gauge fields that couple to the fermionic degrees of freedom with the minimal coupling typical of gauge theories [2].

Another important example of Dirac fermions is that of topological insulators. These materials are characterized by a bulk insulating behavior with conducting boundary states protected by some topological property, the integer quantum Hall effect [3, 4] being the prototype. In the recent developments (for a review, see Refs. 5–7) numerous systems have been found where the edge states exist in the absence of a magnetic field and are protected by time reversal symmetry. The simplest example is the spin Hall effect [8, 9] where the spin orbit coupling plays a major role; most of the newly discovered topological insulators are based on materials with strong spin orbit coupling [10–12]. The topological protection of edge states ensures their stability against non-magnetic – more generally, time-reversal invariant – perturbations, opening up the possibility for future spintronic devices.

Novel exotic states appear at the surface of a three dimensional (3D) topological insulator when an energy gap is induced by a perturbation breaking the time reversal symmetry. They are obtained typically by applying a magnetic field perpendicular to the surface what gives rise to the half integer quantum Hall effect [13] or by proximity effects with a magnetic material what induces an anomalous quantum Hall effect [14]. A very recent experiment reports the observation of massive edge

Dirac fermions in a three dimensional topological insulator doped with magnetic impurities [15]. In these cases the exotic physics is due to a combination of the regular or space-dependent mass induced by the perpendicular magnetic field to the Dirac fermions, and by the appearance of zero modes associated to the Dirac operator in magnetic fields [16–18].

A different situation arises when the surface of the topological insulator is coated with an insulating ferromagnet with an easy plane such that the magnetization is parallel to the surface. Although time reversal invariance is also broken and edge states are no more protected, a mass is not directly induced in the system. In this work we will see that in such situation gauge fields appear coupling to the electronic degrees of freedom similar to these produced by elastic deformation on graphene. We will explore the modification of the spectrum for the cases of a vortex and a skyrmion configuration.

GAUGE FIELD INDUCED BY AN IN-PLANE MAGNETIZATION

When putting a thin ferromagnetic layer with magnetization \vec{M} on the surface of a strong topological insulator an interaction term of the form $H' = -J\vec{M}\cdot\vec{\sigma}$ arises where J is the coupling constant. There are two contributions to this magnetic term: the Zeeman energy, and $s-d$ exchange interaction energy. The latter contribution can be orders of magnitude higher than the former one if the overlap between the electron wave function on the surface of the topological insulator and that of the ferromagnet is big enough. One can easily expect that $JM \sim 0.01-0.1$ eV. This term breaks the time reversal symmetry. In the case of an easy-axis ferromagnet with the magnetization perpendicular to the layer the magnetic interaction has a structure of a mass term proportional to σ_z in the Dirac Hamiltonian resulting in the gap opening mentioned above. In the case of having an easy-plane ferromagnet, the vector \vec{M} lies in the surface plane and

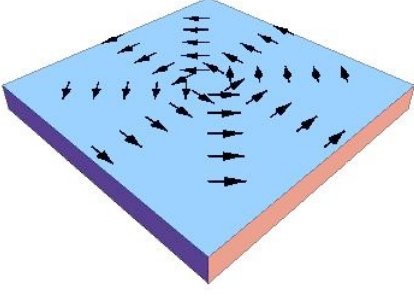


FIG. 1: (Color online) Sketch of the ferromagnetic vortex configuration.

the $s-d$ exchange interaction couples to the fermions as a vector potential.

In the case of graphene where the “spin” degree of freedom is actually a *pseudospin* related to the sublattice degree of freedom, gauge couplings are induced by strains or other mechanical deformations of the lattice [2]. In topological insulators where spin is the real spin one needs in-plane magnetization to create such terms.

Easy plane ferromagnetism is very natural for thin films of single layers as the shape anisotropy (magnetic dipole–dipole interaction) always favor this situation [19]. This will be the case for any soft magnetic material i. e. materials fulfilling the condition $4\pi M_s^2 > K$, K being the constant of magnetocrystalline anisotropy. If the coating is a circular symmetric disc, the magnetization vector will be freely rotating in the plane. Magnetic textures at the surface of a three dimensional topological insulator have been recently studied in Refs. 20, 21. In the latter publication, the exchange field energy associated to parallel ferromagnetic domains of 50 nm width was estimated to be of the order of 6 meV.

Vortex configuration

In easy-plane ferromagnets vortices can exist which are spin configurations with a net 2π twist about a particular point or core similar to the Onsager-Feynman vortices in superfluid helium or Abrikosov vortices in superconductors. Examples are BaCoAsO₄ and K₂CuF₄ [22].

In this case the magnetization is constant in magnitude and rotates in direction as shown schematically in Fig. 1. The corresponding vector potential in polar coordinates is $A_r = 0, A_\theta = -JM$.

Let us assume, for simplicity, that without interaction with magnetization the electron spectrum corresponds to that of isotropic (in plane) Dirac cone. Then, the dynamics of the low energy states of the system in the presence of the vortex is described in polar coordinates by the Hamiltonian

$$\mathcal{H} = i \begin{pmatrix} 0 & e^{-i\theta} (\partial_r - i\frac{\partial_\theta}{r} + JM) \\ e^{i\theta} (\partial_r + i\frac{\partial_\theta}{r} - JM) & 0 \end{pmatrix}. \quad (1)$$

where we have put $\hbar = 1$. From now on and since we will not deal with interactions which can renormalize the Fermi velocity [23] we will put it just to unity: $v = 1$.

The usual ansatz

$$\Psi_1(r, \theta) = e^{il\theta} \varphi_1(r) \quad , \quad \Psi_2(r, \theta) = e^{i(l+1)\theta} \varphi_2(r), \quad (2)$$

allows to write the Schrodinger equation $H\Psi = E\Psi$ with $\Psi = (\Psi_1, \Psi_2)^\dagger$, as

$$\begin{aligned} i \left(\frac{d}{dr} + \frac{l+1}{r} + JM \right) \varphi_2 &= E \varphi_1 \\ i \left(\frac{d}{dr} - \frac{l}{r} - JM \right) \varphi_1 &= E \varphi_2, \end{aligned} \quad (3)$$

which can be reduced to

$$\left[\frac{d^2}{dr^2} + \frac{l}{r} \frac{d}{dr} - \frac{l(l+1)}{r^2} + \frac{JM(2l+1)}{r} \right] \varphi = [(JM)^2 - E^2] \varphi. \quad (4)$$

for both components.

Equation (4) is formally identical to the Schrödinger equation of a planar hydrogen atom with a nucleus of charge Ze [24, 25]:

$$\left(\Delta - 2m \frac{Ze^2}{r} \right) \varphi = -2m\epsilon \varphi, \quad (5)$$

with the substitution $\epsilon = E^2 - (JM)^2$, $2mZe^2 = JM(2l+1)$. From this formal analogy we can write the answer for the eigenvalues of the energy:

$$E^2(n_r, l) = (JM)^2 \left[1 - \frac{(l+1/2)^2}{(n_r + l + 1/2)^2} \right], \quad (6)$$

with $n_r = 0, 1, 2, \dots$

To ease the readout of the spectrum we show a plot of the numerical solution of eq. (1) in Fig. 2 (center). The results reproduce the analytical spectrum given in eq. (6) but only for one sign of the angular momentum. There are no bound states for the other sign. For comparison we also plot the numerical results for the spectrum of a disc as a function of the angular momentum in the presence of a constant magnetic field (Fig. 2 (a)).

We can see that for $2l+1 > 0$ (assuming $JM > 0$) there are localized states in the energy interval $|E| < JM$ with accumulation points at the boundary values $E = \pm JM$. The numerical solution shows also that the wave functions for the localized states decay exponentially at large distances. For any value of l there is a zero-energy mode with $n_r = 0$.

For $2l+1 < 0$ we have the effective Schrödinger equation for the Coulomb center with repulsion, and no localized states.

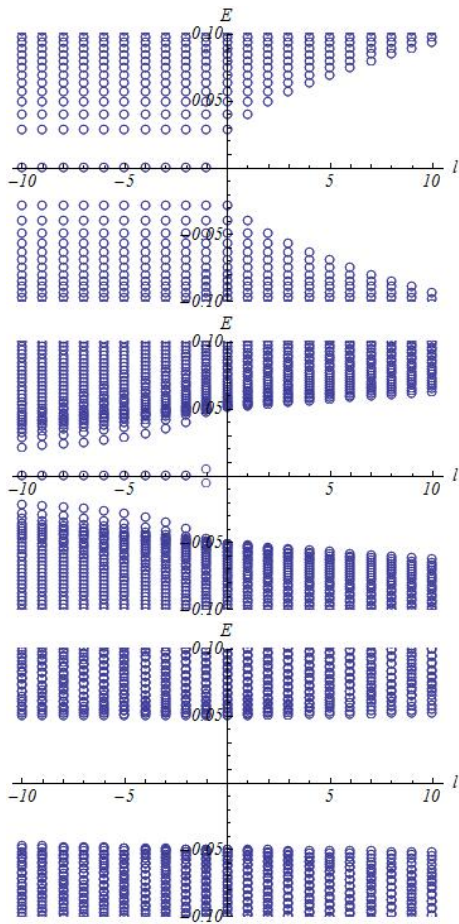


FIG. 2: Top: Energy levels as function of angular momentum at the surface of a topological insulator in a constant in-plane magnetic field. Center: Energy levels in the presence of a magnetic vortex. Bottom: Energy levels in the presence of a skyrmion configuration.

The natural spatial scale in the problem is $\xi = \frac{\hbar v_F}{|J|M}$ which plays the same role as the correlation length for the Abrikosov vortices.

Skyrmion configuration

A novel situation arises when the (3D) magnetic coating is in a skyrmion configuration [26]. 2D skyrmion patterns can exist when the magnetic anisotropy is weak, and they have been recently observed with Lorentz transmission electron microscopy in a thin film of $\text{Fe}_{0.5}\text{Co}_{0.5}\text{Si}$ [27]. The magnetization vector evolves from pointing along a given direction at the center of the defect to the opposite direction at large distances. The absolute magnetization does not change, and at intermediate distances it has an in-plane component. A possible skyrmion tex-

ture is

$$\mathbf{M}(r, \theta) = (\sin f(r) \cos \theta, \sin f(r) \sin \theta, \cos f(r)), \quad (7)$$

where $f(r)$ can be chosen as a constant for simplicity.

A numerical solution of the spectrum induced by a skyrmion texture is shown in Fig. 2 (bottom). The calculation solves an effective one dimensional differential equation for each value of the angular momentum [28]. The figure also shows spectra for a constant magnetic field (top), and for the gauge field associated to a vortex (center). The skyrmion opens a gap in the spectrum and does not support a zero energy solution. Note that a zero mode appears when the in-plane component of the magnetization is subtracted.

CONCLUSIONS AND DISCUSSION

This work originated on the question of whether an effective gauge field would be induced by strain on the surface of a topological insulator. As it is known this is the case in graphene which can be seen in many respects of a precursor of the actual topological insulators. It is known that elastic deformations and geometrical corrugations in graphene can be described by and induced fictitious gauge field coupling to the electronic degrees of freedom [2]. The apparent paradox that the strains do not break time reversal symmetry while magnetic fields do is solved in the graphene case by the fact that the induced fictitious field couples to the two Dirac cones with opposite signs and time reversal symmetry is preserved in the complete system. This mechanism will probably occur also in the case of weak topological insulators with an even number of Dirac cones at the surface. The response of a strong topological insulator to elastic deformations has to be different and remains to be studied. In this work we have seen an alternative way to generate gauge fields in a strong topological insulator and we have studied the modification of the energy spectrum caused by the gauge fields.

Magnetic coating of the surface three dimensional TI is often invoked to describe specific physical situations where a gap is needed in the 2D system [29–31]. The discussion presented in this work shows that in specific situations the magnetic coating can give rise to a strong reorganization of the spectrum that has to be taken into account for each physical situation.

ACKNOWLEDGMENTS

MAHV thanks A. Cortijo and A.G. Grushin for discussions. FG and MAHV acknowledge support from MEC (Spain) through grant FIS2005-05478-C02-01, PIB2010BZ-00512 and CONSOLIDER CSD2007-00010,

and by the Comunidad de Madrid, through CITEC-NOMIK, CM2006-S-0505-ESP-0337. MK acknowledges support from Stichting voor Fundamenteel Onderzoek der Materie (FOM), the Netherlands.

-
- [1] A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim, *Rev. Mod. Phys.* **80**, 315 (2008).
 - [2] M. A. H. Vozmediano, M. I. Katsnelson, and F. Guinea, *Phys. Reports* **496**, 109 (2010).
 - [3] K. von Klitzing, G. Dorda, and M. Pepper, *Phys. Rev. Lett.* **45**, 494 (1980).
 - [4] T. Ando, A. Fowler, and F. Stern, *Rev. Mod. Phys.* **54**, 437 (1982).
 - [5] J. Moore, *Nature Physics* **5**, 378 (2009).
 - [6] X. Qi and S. Zhang, *Phys. Today* **January Issue**, 33 (2010).
 - [7] M. Z. Hasan and C. L. Kane, *Rev. Mod. Phys.* **82**, 3045 (2010).
 - [8] C. Kane and E. J. Mele, *Phys. Rev. Lett.* **95**, 226801 (2005).
 - [9] B. A. Bernevig and S. Zhang, *Phys. Rev. Lett.* **96**, 106802 (2006).
 - [10] B. A. Bernevig, T. L. Hughes, and S. C. Zhang, *Science* **314**, 1757 (2006).
 - [11] Y. Xia, D. Qian, D. Hsieh, L. Wray, A. Pal, H. Lin, A. Bansil, D. Grauer, Y. S. Hor, R. J. Cava, et al., *Nature Physics* **5**, 398 (2009).
 - [12] B. Buttner, C. X. Liu, G. Tkachov, E. G. Novik, C. Brune, H. Buhmann, E. M. Hankiewicz, P. Recher, B. Trauzettel, S. C. Zhang, et al., *Nature Phys.* **6** (2011).
 - [13] L. Fu and C. L. Kane, *Phys. Rev. B* **76**, 045302 (2007).
 - [14] N. Nagaosa, J. Sinova, S. Onoda, A. H. MacDonald, and N. P. Ong, *Rev. Mod. Phys.* **82**, 1539 (2010).
 - [15] Y. L. Chen et al., *Science* **329**, 659 (2010).
 - [16] R. Jackiw and C. Rebbi, *Phys. Rev. D* **13**, 3398 (1976).
 - [17] Y. Aharonov and A. Casher, *Phys. Rev. A* **19**, 2461 (1979).
 - [18] R. Jackiw and P. Rossi, *Nucl. Phys. B* **190**, 681 (1981).
 - [19] S. V. Vonsovsky, *Magnetism, Volume 2* (Wiley, 1974).
 - [20] K. Nomura and N. Nagaosa, *Phys. Rev. B* **82**, 161401 (2010).
 - [21] Y. Zhang and F. Zhai, *Appl. Phys. Lett.* **96**, 172109 (2010).
 - [22] L. P. Regnault and J. Rossat-Mignod, *J. Magn. Magn. Mat.* **14**, 194 (1974).
 - [23] J. González, F. Guinea, and M. A. H. Vozmediano, *Nucl. Phys. B* **424** [FS], 595 (1994).
 - [24] X. L. Yang, S. H. Guo, F. T. Chan, K. W. Wong, and W. Y. Ching, *Phys. Rev. A* **43**, 1186 (1991).
 - [25] V. R. Khalilov and C. Ho, *Mod. Phys. Lett. A* **13**, 615 (1998).
 - [26] T. H. Skyrme, *Proc. Royal Soc. London* **A260**, 127 (1961).
 - [27] X. Z. Yu, Y. Onose, N. Kanazawa, J. H. Park, J. H. Han, Y. Matsui, N. Nagaosa, and Y. Tokura, *Nature* **465**, 901 (2010).
 - [28] B. Wunsch, T. Stauber, and F. Guinea, *Phys. Rev. B* **77**, 035316 (2008).
 - [29] X. Qi, R. Li, J. Zang, and S. Zhang, *Science* **323**, 1184 (2009).
 - [30] J. Alicea, *Phys. Rev. B* **81**, 125318 (2009).
 - [31] A. G. Grushin and A. Cortijo, *Phys. Rev. Lett.* **106**, 020403 (2011).